Hong Kong Mathematics Olympiad (2015/2016)

Final Event 1 (Group)

FOR OFFICIAL USE

Score for accuracy		$\times$	Mult. factor for speed		$=$		Team No.		
				$+$ Bonus score			Time		
				Total score				Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 一項工程包括三個項目：A，B 和 C。若項目 A 開始三天後，項目 B 才可開始進行。項目 C 亦必須在項目 B 開始四天後才可開始進行。若完成項目 A，B 和 C 分別需要四天，六天和五天，求最少天數(P)完成全項工程。

A project comprises of three tasks, A, B and C. Suppose task B must begin 3 days later than task A begins, and task C must begin 4 days later than task B begins. If the numbers of days to complete tasks A, B and C are 4, 6 and 5, respectively, determine the least number of days (P) to complete the project.

P =

2. 指示牌上掛有紅、黃、綠閃燈。紅、黃、綠閃燈分別每隔 3 秒、4 秒、8 秒閃爍一次。當 0 秒時，紅、黃、綠閃燈同時閃爍。若當 Q 秒時，第三次出現只有紅及黃閃燈同時閃爍，求 Q 的值。

There are 3 blinking lights, red, yellow and green, on a panel. Red, yellow and green lights blink at every 3, 4 and 8 seconds, respectively. Suppose each light blinks at the time $t = 0$. At time Q (in seconds), there is the third time at which only red and yellow lights blink, determine the value of Q.

Q =

3. 設

$$f_{n+1}=\begin{cases} f_n+3 & \text{若 } n \text{ 是雙數} \\ f_n-2 & \text{若 } n \text{ 是單數} \end{cases}。$$

若 $f_1=60$，求 n 的最少可能值，令當 $m\geq n$ 時，滿足 $f_m\geq 63$。

Let

$$f_{n+1}=\begin{cases} f_n+3 & \text{if } n \text{ is } even \\ f_n-2 & \text{if } n \text{ is } odd \end{cases}.$$

If $f_1=60$, determine the smallest possible value of n satisfying $f_m\geq 63$ for all $m\geq n$.

n =

4. 求 $T=\left(3^{2^0}+1\right)\times\left(3^{2^1}+1\right)\times\left(3^{2^2}+1\right)\times\cdots\times\left(3^{2^{10}}+1\right)$ 的值。

Determine the value of $T=\left(3^{2^0}+1\right)\times\left(3^{2^1}+1\right)\times\left(3^{2^2}+1\right)\times\cdots\times\left(3^{2^{10}}+1\right)$.

T=

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Final Event 2 (Group)

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			$+$	Bonus score			Time		
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1. 一個盒子有五個球，球面上分別印上號碼 3、4、6、9 或 10。由盒中同時隨機取出 2 個球，並得出其號碼的總和。若 A 為不同總和的數量，求 A 的值。

A box contains five distinctly marked balls with number markings being 3, 4, 6, 9 or 10. Two balls are randomly drawn without replacement from the box. If A is the number of possible distinct sums of the selected numbers, determine the value of A.

A =

2. 設 $f_1 = 9$ 及

$$f_n = \begin{cases} f_{n-1} + 3 & \text{若 } n \text{ 是 3 的倍數} \\ f_{n-1} - 1 & \text{若 } n \text{ 不是 3 的倍數} \end{cases}。$$

若 B 為 k 的值的可能數量，其中 k 滿足 $f_k < 11$，求 B 的值。

Let $f_1 = 9$ and

$$f_n = \begin{cases} f_{n-1} + 3 & \text{if } n \text{ is a multiple of } 3 \\ f_{n-1} - 1 & \text{if } n \text{ is not a multiple of } 3 \end{cases}.$$

If B is the number of possible values of k such that $f_k < 11$, determine the value of B.

B =

3. 設 $a_1, a_2, a_3, a_4, a_5, a_6$ 為非負整數，並滿足

$$\begin{cases} a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 = 26 \\ a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 5 \end{cases}。$$

若 c 為方程系統的解的數量，求 c 的值。

Let $a_1, a_2, a_3, a_4, a_5, a_6$ be non-negative integers and satisfy

$$\begin{cases} a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 = 26 \\ a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 5 \end{cases}.$$

If c is the number of solutions to the system of equations, determine the value of c.

c =

4. 設 d 及 f 為正整數及 $a_1 = 0.9$。若 $a_{i+1} = a_i^2$ 及

$$\prod_{i=1}^{4} a_i = \frac{3^d}{f},$$

求 d 的最少可能值。

Let d and f be positive integers and $a_1 = 0.9$. If $a_{i+1} = a_i^2$ and

$$\prod_{i=1}^{4} a_i = \frac{3^d}{f},$$

determine the smallest possible value of d.

d =

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Final Event 3 (Group)

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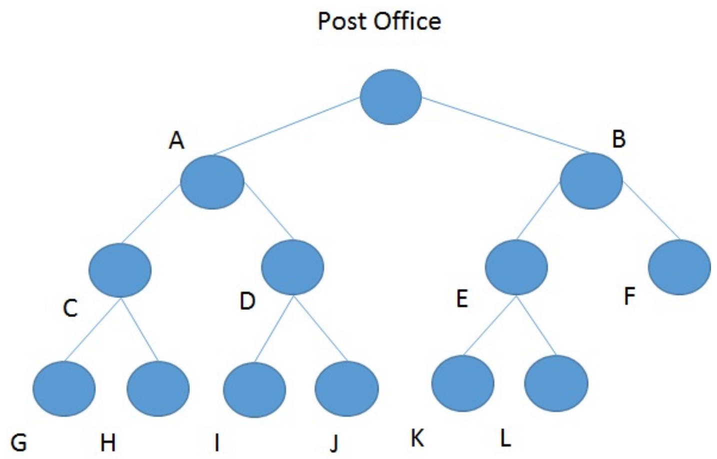
1. 下圖是郵差的送信路線圖：從郵局開始，到達十二個地點送信，最後返回郵局。若郵差從一地點步行到另一相連地點需要十分鐘及 K 為郵差需要的時數來完成整天路線，求 K 的最少可能值。

The figure below represents routes of a postman. Starting at the post office, the postman walks through all the 12 points and finally returns to the post office. If he takes 10 minutes from a point to another adjacent point by walk and K is the number of hours required for the postman to finish the routes, find the smallest possible value of K.



K =

2. 若 n 為正整數，$a_1 = 0.8$ 及 $a_{n+1} = a_n^2$，求 L 的最少值，滿足

$$a_1 \times a_2 \times \cdots \times a_L < 0.3\text{。}$$

If $a_1 = 0.8$ and $a_{n+1} = a_n^2$ for positive integers n, determine the least value of L satisfying

$$a_1 \times a_2 \times \cdots \times a_L < 0.3.$$

L =

3. 若方程 $\sqrt[3]{5+\sqrt{x}}+\sqrt[3]{5-\sqrt{x}}=1$，求實數根 x。

Solve $\sqrt[3]{5+\sqrt{x}}+\sqrt[3]{5-\sqrt{x}}=1$ for real number x.

x =

4. 若 a, b 及 y 為實數，並滿足

$$\begin{cases} a+b+y=5 \\ ab+by+ay=3 \end{cases},$$

求 y 的最大值。

If a, b and y are real numbers and satisfy

$$\begin{cases} a+b+y=5 \\ ab+by+ay=3 \end{cases},$$

determine the greatest possible value of y.

y =

Hong Kong Mathematics Olympiad (2015/2016)
Final Event 4 (Group)

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1. 若 a^2 及 b^2 為整數，且相差 144，求 $d = a + b$ 的最大值。

If a^2 and b^2 are two integers that differ by 144, determine the largest possible value of $d = a + b$.

d =

2. 若 n 為整數，n^2 的個位數及十位數分別為 u 及 7，求 u 的值。

If n is an integer, and the unit and the tens digits of n^2 are u and 7, respectively, determine the value of u.

u =

3. 求實數

$$c=\frac{\left(4+\sqrt{15}\right)^{\frac{3}{2}}+\left(4-\sqrt{15}\right)^{\frac{3}{2}}}{\left(6+\sqrt{35}\right)^{\frac{3}{2}}-\left(6-\sqrt{35}\right)^{\frac{3}{2}}}$$

的值。

Determine the value of real number

$$c=\frac{\left(4+\sqrt{15}\right)^{\frac{3}{2}}+\left(4-\sqrt{15}\right)^{\frac{3}{2}}}{\left(6+\sqrt{35}\right)^{\frac{3}{2}}-\left(6-\sqrt{35}\right)^{\frac{3}{2}}}.$$

$c\ =$

4. 求實數

$$x=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}}}$$

的正數值。

Determine the positive value of the real number

$$x=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}}}.$$

$x\ =$